



7. (a) Halla la transformada de Laplace de la función  $f(t) = \begin{cases} \cos 4t & , \text{ si } 0 \leq t < 4\pi \\ 0 & , \text{ si } t \geq 4\pi \end{cases}$ .

(b) Resuelve el problema de Cauchy:  $\begin{cases} x'' + 16x = f(t) \\ x(0) = x'(0) = 0 \end{cases}$ .

**Sol.:** (a)  $\mathcal{L}(f(t)) = \frac{s(1-e^{-4\pi s})}{s^2+16}$ ; (b)  $x = \frac{t \operatorname{sen} 4t h(t) - (t-4\pi) \operatorname{sen}(t-4\pi) h(t-4\pi)}{8}$ .

8. Resuelve, utilizando transformadas de Laplace, las siguientes ecuaciones diferenciales:

(a)  $\begin{cases} y'' + y = e^t \\ y(1) = 1, y'(1) = 0 \end{cases}$  (b)  $\begin{cases} x'' = 8 - 4t + 2 \operatorname{sen}(2t - 4) \\ x(2) = 1, x'(2) = 0 \end{cases}$

**Sol.:** (a)  $y = \frac{e^{t+(2-e)\cos(t-1)-e\operatorname{sen}(t-1)}}{2} h(t-1)$ ; (b)  $x = [t-1 - \frac{2}{3}(t-2)^3 - \frac{1}{2} \operatorname{sen} 2(t-2)] h(t-2)$ .

9. Resuelve, utilizando transformadas de Laplace, los siguientes sistemas de ecuaciones diferenciales:

(a)  $\begin{cases} x' = 2x - 5y \\ y' = x - 2y \\ x(0) = 1, y(0) = 0 \end{cases}$

(e)  $\begin{cases} x' + 2x + 6 \int_0^t y(u) du = -2 \\ x' + y' + y = 0 \\ x(0) = -5, y(0) = 6 \end{cases}$

(b)  $\begin{cases} x' - 2x + 3y = 4 - 2t \\ y' + 2y - x = 2 - t \\ x(0) = -1, y(0) = 0 \end{cases}$

(f)  $\begin{cases} 9x' - 32y' - 32y = \begin{cases} 0 & , \text{ si } 0 < t < 1 \\ 1 & , \text{ si } t \geq 1 \end{cases} \\ -2x' + \int_0^t x(u) du + 8y' + 8y = 0 \\ x(0) = 32, y(0) = 9 \end{cases}$

(c)  $\begin{cases} x' - y = \begin{cases} 1 & , \text{ si } 0 < t < 1 \\ 0 & , \text{ si } t > 1 \end{cases} \\ y' - x = \begin{cases} 0 & , \text{ si } 0 < t < 2 \\ 1 & , \text{ si } t > 2 \end{cases} \\ x(0) = y(0) = 0 \end{cases}$

(g)  $\begin{cases} x' + y + 3 \int_0^t x(u) du = \begin{cases} 1 & , \text{ si } 0 < t < 1 \\ 0 & , \text{ si } t \geq 1 \end{cases} \\ y' - x = 0 \\ x(0) = y(0) = 0 \end{cases}$

(d)  $\begin{cases} x' - y = \begin{cases} 1 & , \text{ si } 0 < t < 2 \\ 0 & , \text{ si } t > 2 \end{cases} \\ y' - x' = \begin{cases} 0 & , \text{ si } 0 < t < 2 \\ 1 & , \text{ si } t > 2 \end{cases} \\ x(0) = 1, y(0) = 0 \end{cases}$

(h)  $\begin{cases} x'''' = \begin{cases} 2 & , \text{ si } 0 < t \leq 1 \\ 0 & , \text{ si } 1 < t \leq 2 \end{cases} \\ x + y' = 0 \\ x(0) = x'(0) = x''(0) = x'''(0) = 0, y(0) = 3 \end{cases}$

**Sol.:** (a)  $x = \cos t + 2 \operatorname{sen} t, y = \operatorname{sen} t, t \geq 0$ ; (b)  $x(t) = t - e^{-t}, y(t) = 1 - e^{-t}, t \geq 0$ ;

(c)  $x = \begin{cases} \operatorname{senh} t & , \text{ si } 0 < t < 1 \\ \operatorname{senh} t - \operatorname{senh}(t-1) & , \text{ si } 1 < t < 2, \\ \operatorname{senh} t - \operatorname{senh}(t-1) + \operatorname{cosh}(t-2) - 1 & , \text{ si } t > 2 \end{cases}$

$y = \begin{cases} \operatorname{cosh} t - 1 & , \text{ si } 0 < t < 1 \\ \operatorname{cosh} t - \operatorname{cosh}(t-1) & , \text{ si } 1 < t < 2; \\ \operatorname{cosh} t - \operatorname{cosh}(t-1) + \operatorname{senh}(t-2) & , \text{ si } t > 2 \end{cases}$

(d)  $x(t) = e^t h(t) - (t-2)h(t-2), y(t) = (e^t - 1)h(t)$ ;

(e)  $x = 2 - 3e^{-4t} - 4e^t, y = 4e^{-4t} + 2e^t, t \geq 0$ ; (f)  $x = 32 \cos 2t h(t) + \frac{\operatorname{sen} 2(t-1)}{2} h(t-1),$   
 $y = \frac{9}{5} (e^{-t} + 4 \cos 2t - 2 \operatorname{sen} 2t) h(t) + [\frac{-1}{32} - \frac{1}{40} e^{1-t} + \frac{9}{160} \cos 2(t-1) + \frac{9}{80} \operatorname{sen} 2(t-1)] h(t-1)$ ;

(g)  $x = \frac{\operatorname{sen} 2t}{2} h(t) - \frac{\operatorname{sen} 2(t-1)}{2} h(t-1), y = \frac{1-\cos 2t}{4} h(t) - \frac{1-\cos 2(t-1)}{4} h(t-1)$ ;

(h)  $x = \frac{t^4 h(t) - (t-1)^4 h(t-1)}{12}, y = \frac{(180+t^5)h(t) - (t-1)^5 h(t-1)}{60}$ .

10. Resuelve, utilizando transformadas de Laplace, los siguientes sistemas de ecuaciones diferenciales:

$$(a) \begin{cases} x' = -7x - 6y + t \\ y' = 12x + 10y \\ x(3) = 1, y(3) = -8 \end{cases} \quad (b) \begin{cases} x' - y = \begin{cases} 0 & , \text{ si } 0 < t < 2 \\ 1 & , \text{ si } 2 < t < 3 \\ 0 & , \text{ si } t > 3 \end{cases} \\ y' - x = 1 \\ x(1) = y(1) = 1 \end{cases}$$

**Sol.:** (a)  $x = -8 - 5t - e^{t-3} + 25e^{2(t-3)}$ ,  $y = 9 + 6t + 4e^{t-3} - 39e^{2(t-3)}$ ,  $t \geq 3$ ;  
 (b)  $x = [e^{t-1} - 1 + \cosh(t-1)]h(t-1) - \sinh(t-2)h(t-2) + \sinh(t-3)h(t-3)$ ,  
 $y = [\cosh(t-1) + 2\sinh(t-1)]h(t-1) - [\cosh(t-2) - 1]h(t-2) + [\cosh(t-3) - 1]h(t-3)$ .